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Identification Problem for Groundwater Model

An approximate approach for transmissivity

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SOMMARIO

Gli autori propongono un procedimento approssimato per la risoluzione del problema della identificazione della trasmissività nei mezzi porosi, consistente nel ritenerla costante rispetto al gradiente ma non alla divergenza.

Ne segue che detto problema può essere risolto con lo stesso modello matematico che si adotta per il procedimento diretto (calcolo della piezometria) che peraltro risulta più versatile nei confronti della geometria dell'acquifero e richiede minor tempo macchina rispetto ai procedimenti non approssimati.

L'approssimazione sembra possibile negli acquiferi porosi a permeabilità distribuita abbastanza omogeneamente.

E' riportato un esempio numerico riferito alla falda acquifera alluvionale del basso Esino (Ancona, Italy).

INTERNATIONAL ASSOCIATION FOR HYDRAULIC RESEARCH

IDENTIFICATION PROBLEM FOR GROUNDWATER MODELS
An approximate approach for transmissivity

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Summary
The Authors suggest an approximate approach for the solution of the problem of the identification of transmissivity in porous ground: they suppose transmissivity is steady compared to gradient but not to divergence.
It follows such a problem can be solved with the same mathematic model that is adopted for the direct approach (calculation of piezometry).
Approximation looks possible in sufficiently homogeneous aquifers, permeable because of porousity.
We show a numeric example concerning the groundwater flow of the Low Esino Basin (Ancona - Italy).

Resumé
Les Auteurs proposent un procédé approximé pour la résolution du problème de l'identification de la transmissivité dans les moyens poreux: ils supposent que la transmissivité est constante en comparaison du gradient mais non in comparaison de la divergence.
Il suit que ce problème peut être résolu par le même modèle mathematique qu'on adopte pour le procédé direct (calcul de la piëzometrie).
L'approximation semble possible dans les aquifères suffisamment homogènes, perméables par la porosité.
On rapporte un exemple numérique concernant la nappe aquifère du bas Esino (Ancona - Italie).

INTRODUCTION

The increasing water requirement has made more and more urging the problem of water management and particularly the problem of underground water. So, lately, some mathematic model have been achieved, both to simulate the hydrodinamic of underground water and to value the effects of the diffusion of different substances in it.

One can use such models if:

i - the flow is laminar (see Darcy's law)

ii- the vertical dimension is negligible as to the planimetric ones (it very often takes place in the underground water of the alluvial plains).

So the classic parabolic (bidimensional) equation is true

$$\mathrm{div}\ (T\ \mathrm{grad}\ H) = -q + S\ \frac{\partial H}{\partial t} \qquad (1)$$

where:

T (x,y) is transmissivity (that's the function by which one can measure how easily the water crosses the porous ground):

S (x,y) is the storage coefficient (that's the measure of the water capacity); (dimensionless);

q (x,y,t) is the range of the well or of the linear "mathematic" source; (m/s);

H (x,y,t) is the piezometric height referring to an horizontal plane (m)

The parabolic equation (1) linked with the contour conditions and with the starting ones, discretized at the differences or at the finite elements, is solved by digital methods and gives the answer to some hypothesis of management.

Yet the goodness of such simulations depends on the knowledge of the physical parameters (transmissivity and storage coefficient) which can't be measured directly but only evaluated (Rushton 1978; Sondhi and Singh 1978 to mention only the latest works on the subject) by tests of range, concerning, of course, a very few difficultly extrapolable points of the domain.

Another method to get the values of the physical parameters is to calculate them by (1), as if it were an inverse problem.

Several authors such as Scarascia and Ponzini (1972), Galligani and Valente (1975), Cooley (1977), Guvanasen and Volker (1978) studied it, but from a mathematical point of view, the problem isn't partially solved, because the unicity of the inverse problem solution hasn't in general been proved.

However the method has spread as "la sperimentazione fatta tende a dimostrare che è possibile giungere ad una valida determinazione del parametro trasmissività se si conosce una buona stima di tale parametro" (VALENTE 1978).

A SUGGESTED APPROXIMATION

With this work we aim at solving such a problem as far as it concerns the evaluation of transmissivity, which permits to study only the case of permanent motion and (1) turns into:

$$\mathrm{div}\ (T\ \mathrm{grad}\ H) = -q \qquad (2)$$

which results easily integrable into H, but not into T.

Let's suppose that T is steady compared to gradient but not to divergence or (that's the same) that:

$$\frac{\partial T}{\partial x} \quad \text{and} \quad \frac{\partial T}{\partial y}$$

are very small, that's transmissivity changes very gradually, it follows as

$$T\ \frac{\partial H}{\partial x} \simeq \frac{\partial (T\ H)}{\partial x} \quad \text{and} \quad T\ \frac{\partial H}{\partial y} \simeq \frac{\partial (T\ H)}{\partial y} \qquad (3)$$

and (2) turns into

$$\nabla^2\ (T\ H) = -q \qquad (4)$$

which is easily integrable into it and can be solved by the same methods as used for the solution of (2) into H.

The approximations are not always possible, of course: in fact one must change transmissivity very gradually, but it is sufficiently verified (at least terrace by terrace) in the alluvial aquifers.

Moreover researches of geological and/or geoelectric character can give very useful information whether the suggested approximation can be carried out or not.

It seems moreover, at least for what we have until now tried, that it is enough (by pumping-test) to know a few points, even internal of the basin, to make the problem determinate.

EXEMPLIFICATION

The mathematic model, by finite differences method, which was performed in the Institute of Applied Geology (MANTICA 1979) to solve (1) was carried out to integrate (4).

A control parameter has been introduced into it to change the calculation of the matrix coefficients (of the linear system ensuing from discretization) according to the way of proceeding, which can be either a direct type (piezometric calculation) or an identification one.

The model, modified like that, has the logic scheme of fig. 1 and has been applied to the Middle and low Esino Basin about which lots of observations giving a fairly precise knowledge of piezometry (fig. 2), were available (CRESCENTI & others 1978).

On the contrary, the data of the water withdrawal from the wells and of the infiltrations, about which researches are still taking place, are less reliable.

In this work the difference vector infiltrations-pumpings has been reckoned by the few available data and it cannot be considered reliable with the same precision adopted for piezometry.

To be able to make comparisons, which are spoken in the next paragraph and to limit to the terrace of IV order, to support the verification of (3), the domain for which the calculation has been made is the only one showed in fig. 3, enclosed in the squared rectangle the real side of which are 5 km. x 4 km.

Discretization consists in a square meshed grid the side of which is 500 m. long and parallel to the sides of the rectangle.

CONCLUSION

As above said, the model we have suggested has been applied to the Middle and Low Esino groundwater flow, where we have a good knowledge of piezometry and where the conditions required for such approximation are true.
Fig. 4 and 5 which, rispectively, draw the isopiestic lines taken from "Crescenti and others" and those estimated by the direct model using the transmissivities estimated by the approximate approach we have suggested, show the goodness of the performed results.
The model carried out on the computer CDC 7600 from N.E. Italy Interuniversity Calculation Centre (Casalecchio di Reno - Bologna) requires, for the area taken into consideration, a 5 seconds' calculation time, which can be considered sufficiently short, by now.
At present comparisons between the model, we have suggested, and those requiring transmissivity as a solution of the inverse problem are taken place, in order to be able to give a definitive judgement on the goodness and on the advantage coming out of a specified type of exemplification, if the conditions to apply it come true.

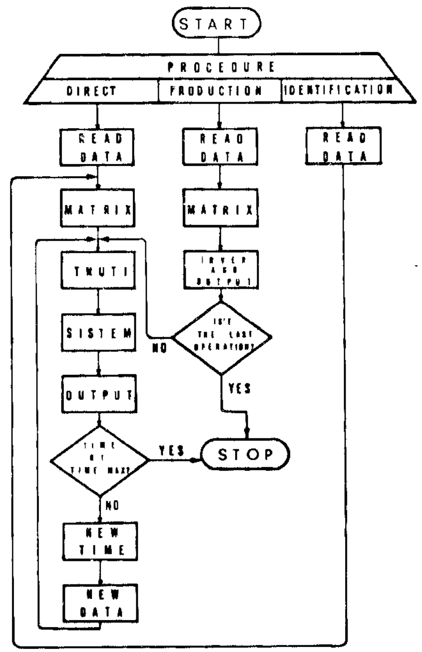

FIG. 1
SINTETICAL FLOWCHART OF MATHEMATICAL MODEL
FLOWCHART SYNTHÉTIQUE DU MODÈLE MATHÉMATIQUE

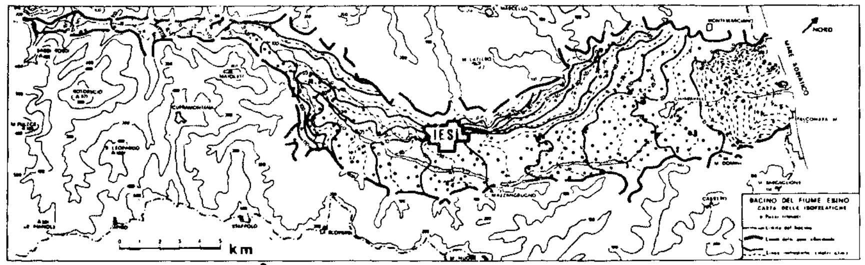

FIG. 2 - MEASURED ISOPIESTIC LINES OF THE MIDDLE AND LOW ESINO BASIN.
ISOPIÈZES MESURÉES DU BASIN DU MOYEN ET BAS ESINO.

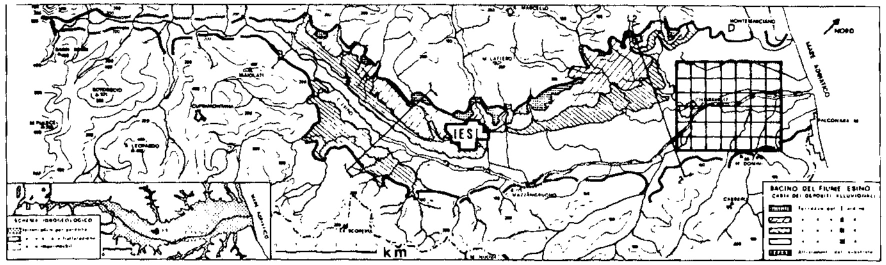

FIG. 3 - THE MIDDLE AND LOW ESINO BASIN AND FIELD OF MODEL.
BASSIN DU LE MOYEN ET BAS ESINO ET CHAMP DU MODÈLE.

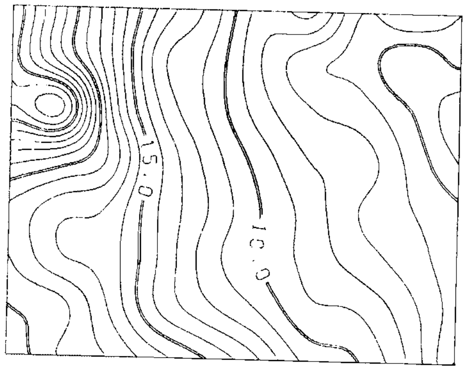

FIG. 4 - ISOPIESTIC LINES (MEASURED).
ISOPIÈZES MESURÈES.

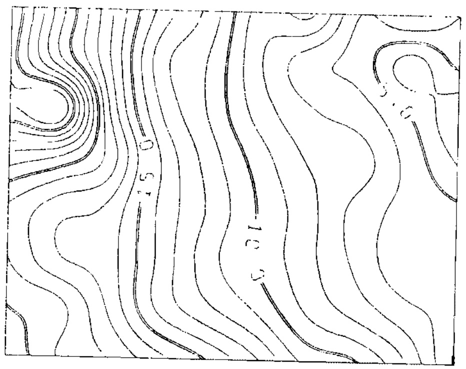

FIG. 5 - ISOPIESTIC LINES (CALCULATED).
ISOPIÈZES CALCULETE.

BIBLIOGRAPHY

COOLEY (1977) - A method of estimating parameters and assensing reliability for models of steady state groundwater flow, 1, Theory and Numerical properties. WRERAQ (13-2).

CRESCENTI, MEDI, NANNI, BRUNI, CALVI, GOBBI, FOGLIA, Renga (1978) - La falda acquifera del subalveo del medio e basso Esino. Istituto di Geologia Applicata dell'Università di Ancona.

GALLIGANI (1974) - Identification problems and modelling. Pubblicazione n. 96 serie III dell'Istituto di Applicazione del Calcolo del C.N.R. - Roma.

GALLIGANI and VALENTE (1975) - Identificazione dei modelli per lo studio delle acque sotterranee Applicazioni del Calcolo - MARVES - Roma.

CHVANASEN and VOLKER (1978) - Identification of distributed parameters in groundwater basin. Jr. of Hydrology Vol. 36 n. 3/4 February.

MANTICA (1979) - Un codice di calcolo multiuso per lo studio della dinamica delle acque sotterranee. Rapporto interno. Istituto di Geologia Applicata. Fac. Ingegneria. Ancona. (di prossima pubblicazione).

RUSHTON (1978) - Estimating transmissivity and storage coefficient from abstraction well data. Groundwater. Vol. 16 n. 2 March/April.

SCARASCIA and PONZINI (1972) - An approximate solution for the inverse problem in hydraulics. L'Energia elettrica. Vol. XLIX n. 8.

SONDHI and SINCH (1978) - Determination of unconfined aquifer parameters using partially penetrating wells. Jr. of Hydrology Vol. 36 n. 3/4 February.

VALENTE (1978) - Un codice di calcolo per l'identificazione dei parametri di trasmissività e di immagazzinamento in acquiferi sotterranei. Pubblicazione n. 64 serie III dell'Istituto di Applicazioni del Calcolo del C.N.R. - Roma.

GIURA (1975) - Modelli matematici per lo studio di un sistema di risorse idriche sotterranee. Giornate di studio "Problemi di modellistica e controllo nella gestione delle risorse idriche" Università della Calabria (Arcavata).